Apropos THINK GLOBALLY, and helical paths on tin cans. Slide rules have been crowded out by pocket calculators, which is a pity. During my travels I once purchased a circular slide rule which is in a way better than the straight one, e.g., when we have to multiply several numbers the result overflows the scale on the right, or when we have to perform serial division and it overflows on the left. When it is a mix of serial multiplications and serial divisions, we could overflow either end of the scale on the straight, but not on the circular slide rule. I have never come across an actual copy of the greatest slide rule, which would be one helix sliding along another of the same dimensions. It would be the most efficient and most compact slide rule ever, if it was constructed. Before I could patent my invention, the computer revolution put me out of business.

But what interests the differential geometer is that these three are the ONLY possibilities to construct slide rules. To prove this is as simple as it can be. There are only three self-sliding space curves, because they must have constant curvature and constant torsion. If both the curvature and the torsion are 0, we have the straight slide rule. If the torsion is 0 and the curvature is non-zero, we have the circular slide rule (incidentally, the radius of the circle is just the reciprocal of the curvature!). If both the curvature and the torsion are non-zero, we have the helical slide rule. The curvature is the numerical measure of deviation from a straight line; the torsion is the numerical measure of deviation from a plane curve. In general, they both vary along the flypath. The best way to visualize this is to think that we are flying in a space where there is no gravitation. Our plane has only two control knobs: one to control curvature and the other, torsion. If we lock both, the flypath will be a helix!

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Apropos the shortest distance between two points on a curved surface, the idea of minimal surfaces is a generalization. Here a contour curve is given which is a closed loop, and we are looking for the surface with the smallest area resting on that contour. Clearly, this is a generalization of the shortest path problem. The interesting thing is that in the general case there is only one solution, just as in case of finding the shortest path where the role of the contour is played by a pair of points. I say in the general case, because in some special cases uniqueness may disappear, just as on the globe when the pair of points happens to be antipodal (e.g., the North and South poles) when there are infinitely many shortest paths, namely great circles (meridians).

Incidentally, the problem of minimal surfaces is a hard one. It involves solving a second order partial differential equation. When, several decades ago, I was teaching differential geometry the last time, it took a long time for the best computers available to solve those differential equations. Maybe today they could do it much faster, I don’t know. But the solution would not be instantaneous. Well, ours are digital computers, and we are fond of disparaging analog computers. Yet nature runs analog computers exclusively, and minimal surfaces offer a shining example how efficient they are. Fashion the contour out of a wire loop, attach a handle to it, and dip it into a soapy solution. When you pull it out, bingo! The minimal surface appears as a film inside of the contour! The solution is instantaneous (no pun intended: I don’t mean the soapy solution; I mean solution to the differential equation!), and if we find a way to manipulate the contour loop, then we can follow the change of the shape of the minimal surface as a function of the contour.
I posted a blog to the SQUARE DANCING article of Professor Strogatz under the title SQUARE DANCING ON SQUARE TILES. Unfortunately, my blog was misplaced and ended up as blog #268 attached to the article DIVISION AND ITS DISCONTENTS to which it is not relevant. Also, I posted a blog to the article FINDING YOUR ROOTS under the title FROM RATIONAL TO IRRATIONAL. It is in answer to Settembrini’s thoughtful comment #237 on my blog #230. In any case, I am publishing my blogs separately (preferably in an expanded version) on my own website: www.professorfekete.com/mathematics/blogs to avoid future mishaps.