Chapter 1: INTRODUCTION

For 400 years logarithms were the mainstay of mathematics. By now they have all but disappeared from the school curriculum. This is a pity. Logarithms were conceived as a method to simplify calculations. The logarithm of a *product* is the *sum* of logarithms; that of a *fraction* is the *difference*. Just as multiplication and division are reduced to addition and subtraction, raising to a power and root extraction are reduced to multiplication and division. In particular, square roots are reduced to division by 2; cube roots, division by 3.

It is true that our pocket calculators, not to mention our electronic computers can do more, and do it more quickly. Still, a great deal has been lost with logarithms, especially about the mechanism of simplifying calculation. We no longer understand (or are curious) why a particular type of simplification *does* work. We are no longer in command. Computers *are*.

Stepnumbers are designed to do calculations with extremely large numbers, the decimal representations of which have an inordinate number of digits, counted in the trillions, quadrillions, quintillions, etc., beyond the capability of the memory units of computers to handle them. The 'milestones' of the stepnumber system are the famous Bell numbers: **1** = 1, **10** = 2, **100** = 5, **1000** = 15, **10000** = 52, **100000** = 203, **1000000** = 877, **10000000** = 4140, **100000000 =** 21147, **1000000000** = 115975, … The stepnumber system works with infinitely many digits, but it is most economical with the introduction of a new digit. It waits to the last moment. When this introduction can no longer be postponed, the new digit appears but once. A second and third appearance has to wait almost as long. Only after its fourth appearance will the new digit appear more frequently.

It is very likely that with the further development of science much larger numbers will be needed than those that have been used so far. It is true that, unlike the Romans, we *can write* any large number with the aid of number systems we already have. For example, a number with 44 decimals falls between 10^{44} and 10^{45} and 10^n can be made arbitrarily large by making n sufficiently large. However, the ability to write down a very large number is one thing, and the ability to do calculations with it is another. The stepnumber system makes accurate calculations possible with those very large numbers that are beyond the reach of the present generation of computers.

As an example of the need for ever larger numbers and the need to do calculations with them consider the infinite sequence of prime numbers 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101,…, the building blocks of the natural numbers and their extensions, rational and real numbers. Prime numbers have important applications in chemistry and chrystallography, among other branches of science. A field of application where the need to have ever more of them is pressing is cryptology, the science of security codes. In order to improve security, there is need for ever larger prime numbers, to keep ahead of code-breakers and other hackers.

In 1876 the largest prime number known took 44 decimal digits to write, a record that stood for 75 years. In 1951 electronic computers were put in the service of finding ever larger prime numbers. Today, the largest known prime number takes almost 13 million decimal digits (!) to write, and it has been predicted that by 2024 the largest known prime number will take 1000 million decimal digits. In the past, similar predictions were pretty accurate. They may or may not be in the future. But it gives us some idea and warning: at this pace we shall run out of computer memory capacity, especially as the binary number system, the staple language of computer calculation, is the most uneconomical of all number systems. It takes more digits to write down a number in binary form than in any other.

Another possible application of stepnumbers is the transliteration of Chinese characters. Should the Chinese ever decide to reform their system of writing, they could use stepnumbers instead of characters. Frequently occurring characters could be substituted by stepnumbers using only the first few digits. Characters that occur less frequently would then be replaced with stepnumbers using several digits. With nine digits the whole spectrum of Chinese characters could be covered, with plenty of stepnumbers left over to represent characters that haven't come along yet.

This book reproduces the notes I have used in my interactive lectures to 7-15 years old pupils. The only prerequisite was that the pupils had to be familiar and conversant with the four arithmetic rules: addition, multiplication, subtraction and division, as well as inequalities. We faced the problem of inventing infinitely many digits. So we colored the available decimal digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 with ten monochromatic colours of the rainbow, the so-called *basic colours*: infrared (black), red, brown, orange, yellow, green, viridian (a colour between green and blue), blue, indigo (a colour between blue and violet), violet. In this way we got the first one hundred digits of the stepnumber system:

0123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890123456789

To get more digits, we proceeded as follows. The frequencies of the basic colours divide the spectrum into segments. Each segment contains various shades between adjacent basic colours. These frequencies are very large numbers, but this is a consequence of our choice for the unit of time, the second, that is relatively very large. By a judicious change to a smaller unit we can make the frequencies of the basic colours consecutive integers: 16 for infra-red, 17 for red, 18 for brown,…, 25 for violet, as shown by the table:

infrared	red	brown	orange	yellow	green	viridian	blue	indigo	violet
16	17	18	19	20	21	22	23	24	25
16-17	17-18	18-19	19-20	20-21	21-22	22-23	23-24	24-25	25-26

which also shows the ranges of fractional frequencies between the consecutive integers from 16 through 25 for the various shades between the basic colours. We now simply use the colours with fractional frequencies to paint the decimal digits to get infinitely many digits needed for the stepnumber system.

During these interactive lectures we have reinvented logarithms. In fact, we have invented two new types: Cantor's logarithms (motivated by Cantor's number system) and, our main interest, "steplogarithms" (motivated by the stepnumber system). Each of these gives rise to a new type of slide rule, different from the conventional one, but working on the same principle. The slide rule of steplogarithms is appropriately called the *rainbow slide rule*. It has its stationary and sliding scales subdivided into ten equal segments coloured by one of the ten basic colours, arranged in the same order. The scales within each coloured segment are the same. They are neither linear nor logarithmic but are obtained through interpolation from the reciprocal Bell numbers. The advantage of the rainbow slide rule is that it works for extremely large numbers doing multiplication, division, the power operation and root extraction. It also furnishes the steplogarithms of numbers.

The pupils should enjoy working with steplogarithms as well as with the rainbow slide rule. Their understanding of the mechanism simplifying calculations should be greatly enhanced. They would also have a visual appreciation of the stepnumber system.

This is non-conventional mathematics at its best. In the computer age our pupils have been robbed of their opportunity to learn, understand and appreciate much good mathematics that an earlier generation of pupils could take for granted. This pioneering course is designed to compensate them for their loss. At the same time I also hope that this course has also helped to prepare them for a new generation of computers, the so-called *quantum computers* which use the various energy states of atoms for coding purposes.

In more details, quantum computers code the stepnumbers by assigning various admissible orbits of electrons in the atom circling around the nucleus to various rainbow digits. Compare this to the present generation of computers which code the binary numbers by assigning the magnetic poles North and South to the binary digits **0** and **1**.

It can be seen that the stepnumber system fits quantum computers like the glove fits the hand. Together they open a New Age in information science and technology.